LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **JUNE 2015**

MT 3503 - VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date : 01/07/2015 Dept. No. Max. : 100 Marks

Time : 10:00-01:00

PART – A

ANSWER ALL THE QUESTIONS **(10 x 2 = 20)**

1. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\nabla x \vec{r} = \vec{0}.$
2. If the vector $3x\vec{i} + (x+y)\vec{j} - az\vec{k}$ is solenoidal, then find the value of a.
3. What is the necessary and sufficient condition for the line integral to be independent of path of integration ?
4. Define conservative field and scalar potential.
5. State Gauss theorem.
6. State Stoke's theorem.
7. Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0.$
8. Solve $\frac{dy}{dx} + y\cot x = 4x\cos ecx.$
9. Solve $\left(D^2 + 5D + 6\right)y = 0.$
10. Find the particular integral of $\left(D^2 + 4\right)y = Sin2x$.

PART – B

ANSWER ANY FIVE QUESTIONS **(5 x 8 = 40)**

11. If $\nabla\phi = \left(y + y^2 + z^2\right)\vec{i} + \left(x + z + 2xy\right)\vec{j} + \left(y + 2zx\right)\vec{k}$ and if $\phi(1,1,1) = 3,$ find ϕ.
12. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |r|$, show that $\nabla.\left(r^n\vec{r}\right) = (n+3)r^n$
13. Verify Green's theorem in the plane for the integral $\int_C \left(xy + y^2\right)dx + x^2dy,$ where C is the curve enclosing the region R bounded by the parabola $y = x^2$ and the line y = x.
14. Evaluate $\iiint_V \vec{F}.dv,$ where $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$ and V is the volume of the region enclosed by the cylinder $x^2 + y^2 = a^2$ between the planes z = 0, z = c.
15. Solve $xp^2 - yp - x = 0$.
16. Solve $\left(px - y\right)\left(py + x\right) = 2p$.

17. Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}=x+1$.

18. Solve $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^x\cos 2x$

PART – C

ANSWER ANY TWO QUESTIONS **(2x 20 = 40)**

19. (a) In the vector field $\vec{F}=z\left(x\vec{i}+y\vec{j}+z\vec{k}\right)$, evaluate $\int_C \vec{F}.\vec{dr}$ along the following curves.

(i) Curves $x=t, y=t^2, z=t^3$ from (0,0,0) to (1,1,1).

(ii) Rectilinear curve obtained by joining O (0,0,0), A(1,0,0), B(1,1,0), C (1,1,1) by straight lines.

(b) Evaluate $\iint_S \vec{F}.n$ dS where $\vec{F}=\left(x+y\right)\vec{i}+x\vec{j}+z\vec{k}$ and S is the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

20. Verify Stoke's theorem for $\vec{A}=(2x-y)\vec{i}-yz^2\vec{j}-y^2z\vec{k}$ taken over the upper half surface of the sphere $x^2+y^2+z^2=1$, $z\geq 0$ and the boundary curve C, the $x^2+y^2=1$, z = 0.

21. (a) Solve $\frac{dy}{dx}=2y\tan x+y^2\tan^2 x$ given y = 1 when x = 0.

(b) Solve $\left(1+e^x y+xe^x y\right)dx+\left(xe^x+2\right)dy=0$.

22. (a) Solve $\left(x^2D^2-2xD-4\right)y=x^2+2\log x$.

(b) Solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$, using variation of parameters.

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